[Use a separate Answer Book for each Group]

Group – A
(Answer any five questions)   [5×5]

1. Expand $\cos^7\theta$ in a series of cosines of multiples of θ.   [5]

2. Find the condition that the equation $x^3 + px^2 + qx + r = 0$ may have two roots of equal moduli but of opposite signs.   [5]

3. Prove that in an equation with real coefficients, imaginary roots occur in conjugate pairs.   [5]

4. Give the statement of Descartes' rule of sign. Applying this rule, find the nature of the roots of the equation $x^4 + 16x^2 + 7x - 11 = 0$ [2+3]

5. Solve by Cramer's rule : $2x - z = 1,\ 2x + 4y - z = 1,\ x - 8y - 3z = -2$.   [5]

6. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

a) Find the eigen values of A.   [2]

b) Find the eigen values of $\left(A^{-1}\right)^5$ [3]

7. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -5 \end{bmatrix}$$ [5]

8. Verify Caley-Hamilton theorem for the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ [5]

Group – B
(Answer any five questions)   [5×5]

9. Show that $\lim\limits_{x \to 0} \dfrac{e^{1/x}}{e^{1/x} + 1}$ does not exist.   [5]

10. Prove that if a function $f : [a, b] \to \mathbb{R}$ is differentiable at a point $c \in (a, b)$, then it is continuous at c.   [5]

11. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = |x| + |x - 1|^2,\ \forall x \in \mathbb{R}.$$

Determine the continuity and differentiability of f at x = 0 and x = 1.   [5]

12. If the function f is defined on [a,b] and the first order derivative of f exists and be negative at the point $c \in [a,b]$, then prove that the function is decreasing at 'c'. [5]

13. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} = 0$. [5]

14. State the chain rule for the derivation of composite functions. Applying this chain rule, compute $\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$ where $z = e^{xy^2}$, x= t cost, y = t sint. [2+3]

15. Show that the function $f(x,y) = \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, (x,y) \neq (0,0)$

$$= 0 \qquad ,(x,y) = (0,0)$$

does not satisfy the conditions of Schwarz's theorem and $f_{xy}(0,0) \neq f_{yx}(0,0)$ [3+2]

16. State and prove Euler's theorem on homogeneous function, for a function of three variables. [5]

Group – C
(Answer <u>any five</u> questions) [5×5]

17. a) What do you mean by 'central tendency' of a frequency distribution? What are its measures?
    b) Prove that the arithmetic mean is affected by change of both origin and scale. [1+1+3]

18. For any n-positive numbers $x_1, x_2 .... x_n$, show that AM ≥ GM ≥ HM. [5]

19. a) For a discrete variable, show that 'standard deviation' is the least root-mean-square-deviation.
    b) The relation between two variables x and y is $2x - 5y = 10$, and the quartile deviation of x is 5. Find the quartile deviation of y. [3+2]

20. Discuss the advantages and disadvantages of different measures of central tendency. [5]

21. a) Show that first order central moment for any variables is always zero.
    b) In a certain distribution, the first four moments about the value 4 of the variable are 1,4,10 and 45 respectively. Find the moments about mean. [2+3]

22. a) What are the 'skewness' and 'kurtosis' of a distribution?
    b) The first four moments of a distribution about the value 4 of the variable are −1.5, 17, −30 and 108 respectively. Calculate b_1 and b_2. [2+3]

23. a) Show that the correlation coefficient r_{xy} of variables x and y lies within −1 to 1.

    b) Suppose u = cx + dy , v = cx − dy and r is the correlation coefficient between x and y. If u and v are uncorrelated then prove that $s_u s_v = 2cd\, s_x s_y \sqrt{1 - r^2}$. [3+2]

24. Out of two lines of regression given by, $x + 2y - 5 = 0$ and $2x+3y = 8$, which one is regression line of x on y and y on x? Use the equation to find the means of x and y. If the variance of x is 'v', calculate the variance of y. [5]

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